CIKM 2016
Indianapolis
October 24-28, 2016

Derivative Delay Embedding: Online Modeling of Streaming Time Series

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Outline

1. Challenges of Online Modeling
2. Derivative Delay Embedding (DDE)
3. Markov Geographic Model (MGM)
4. Experimental Results

Challenges of Online Modeling

Most modeling methods require pre-processing or assumptions:

- **Segmentation**
- **Alignment**
- **Normalization**

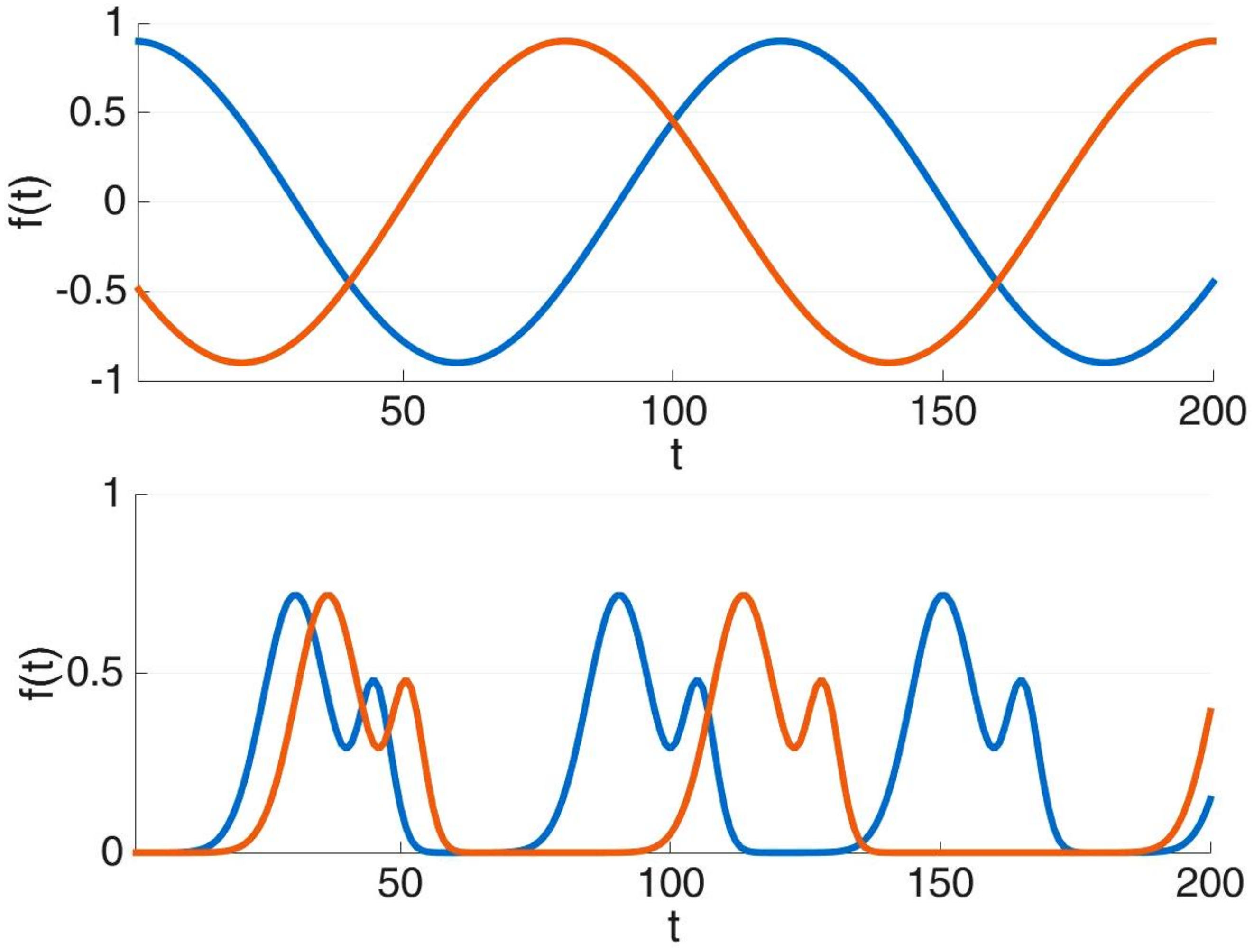
However, for the online scenario:

- **Infinite time series**
- **Real-time**
- **Misalignment**

Pre-processing and unrealistic assumptions are not allowed, thus misalignment challenges the online modeling

Misalignment in Online Modeling

Misalignment mainly refers to the variation in phase and repeat rate of streaming time series.



The Proposed Approach

- Misaligned
- Non-periodic

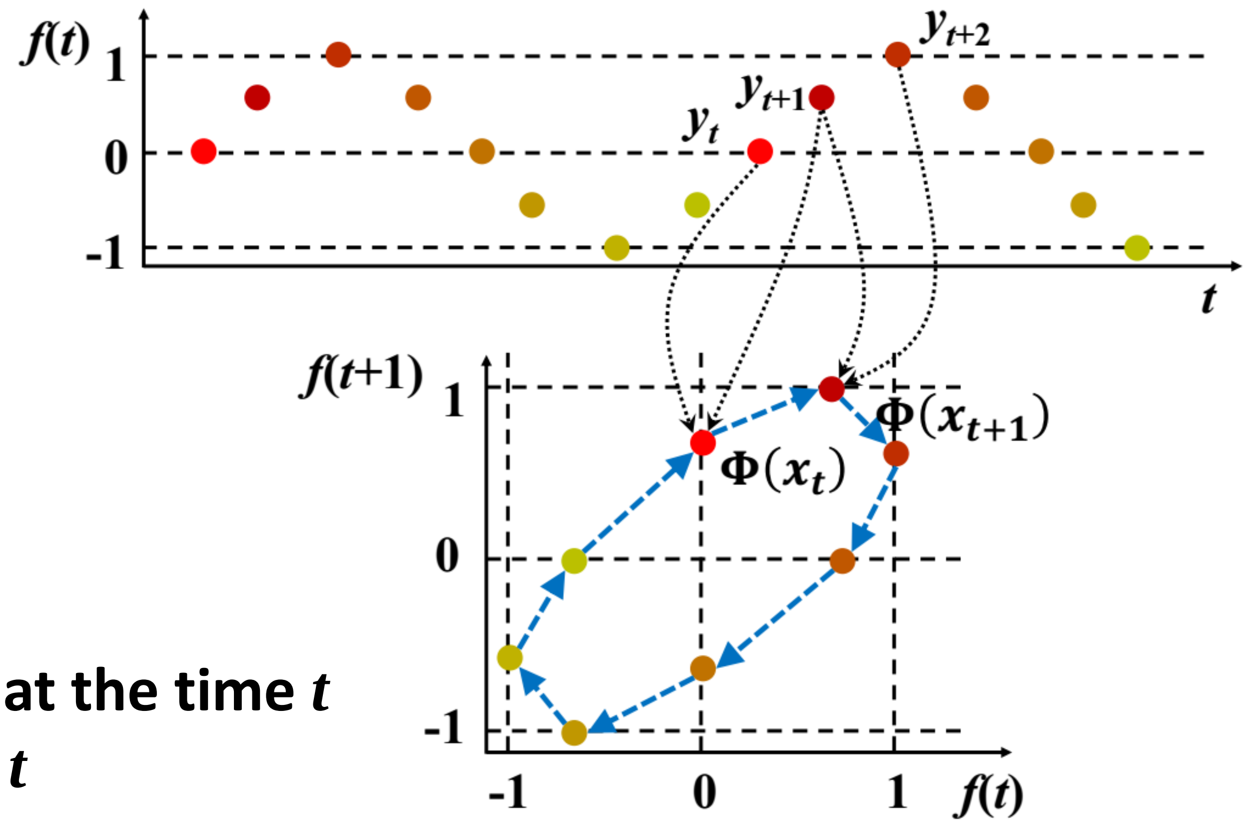
- Invariant to misalignment
- Real-time
- Incremental manner

- Online modeling
- Online testing

Delay Embedding (DE)

Reconstruct a latent dynamical system which generates the time series regardless of misalignment.

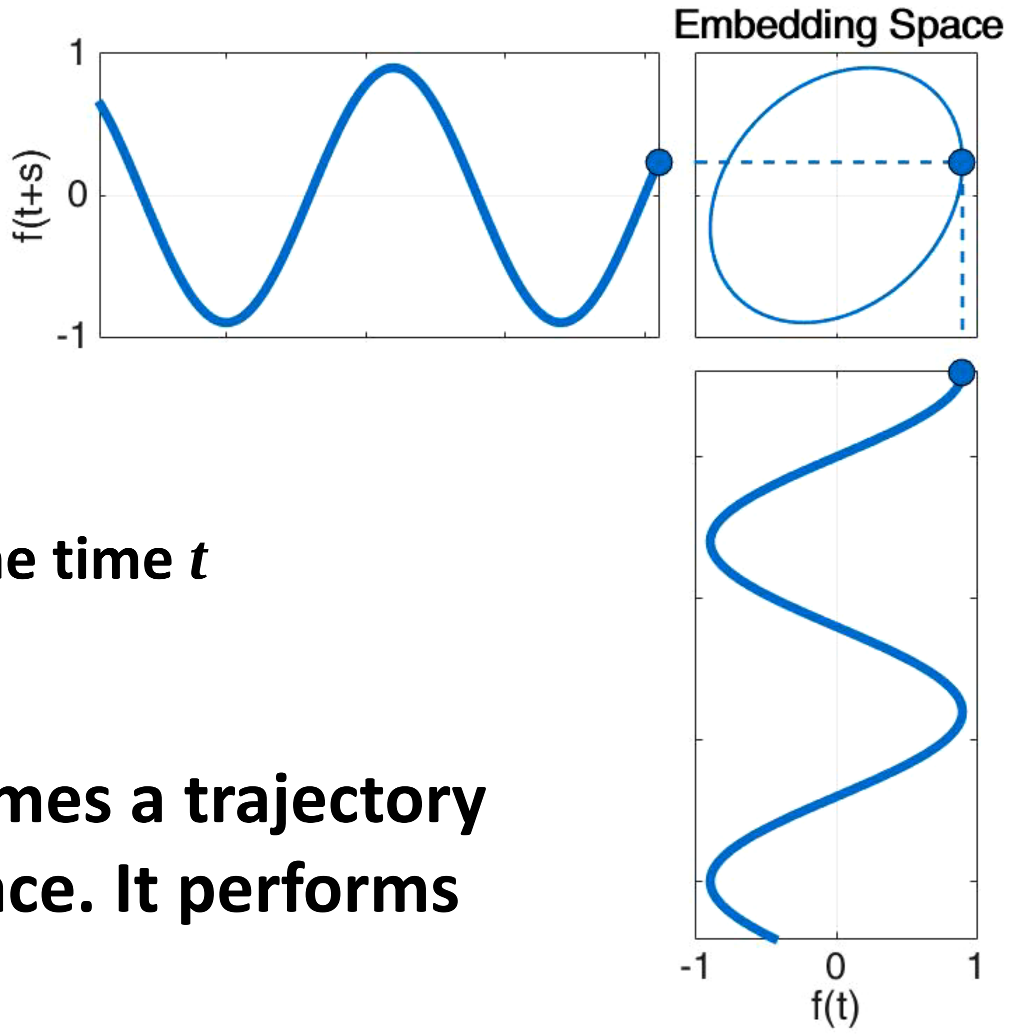
$$\Phi(x_t; s, d) = \left(y_t, y_{t+s}, \cdots, y_{t+(d-1)s}\right)$$



Φ --- estimate of x
s --- delay step
d --- embedding dimension
x_t --- state of the latent dynamical system at the time t
y_t --- observation (time series) at the time t

A Toy Examples of Delay Embedding

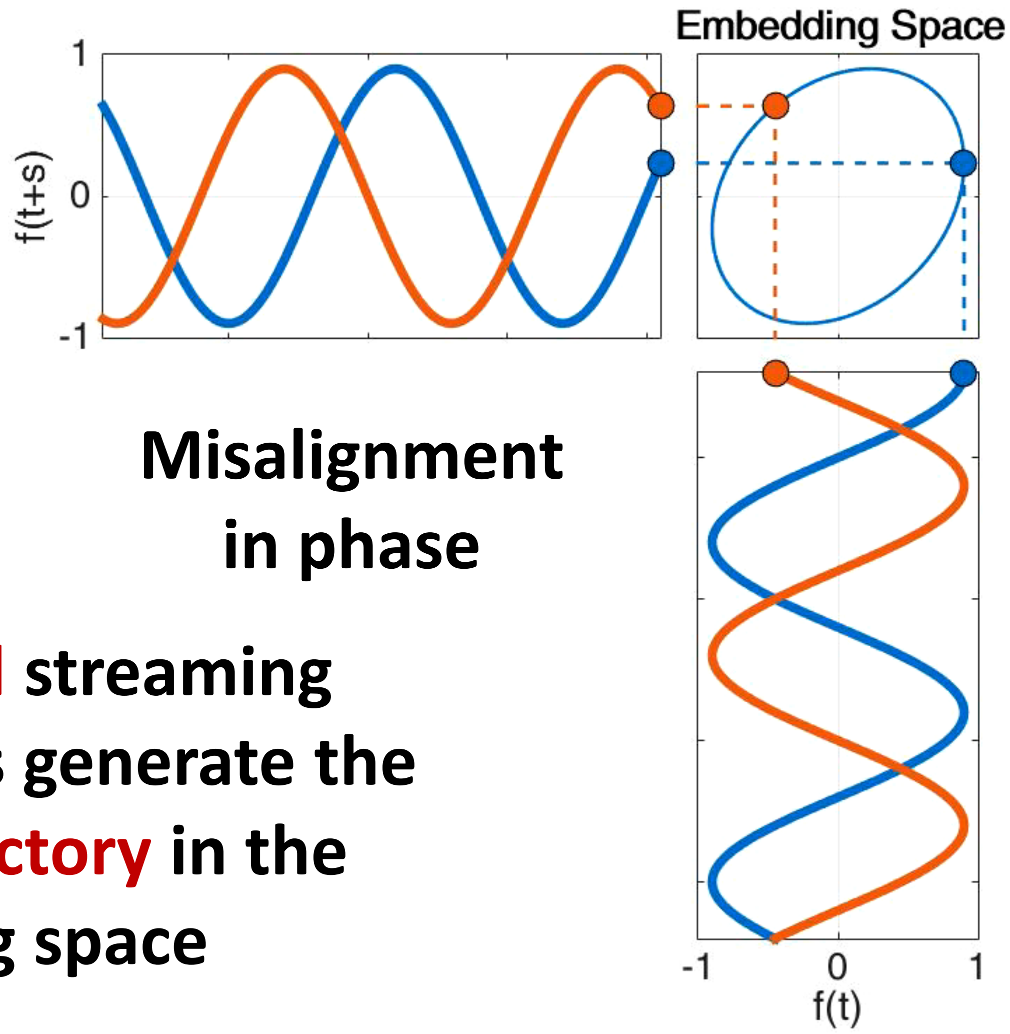
$$\Phi(x_t; s, d=2) = (y_t, y_{t+s}) = (f(t), f(t+s))$$



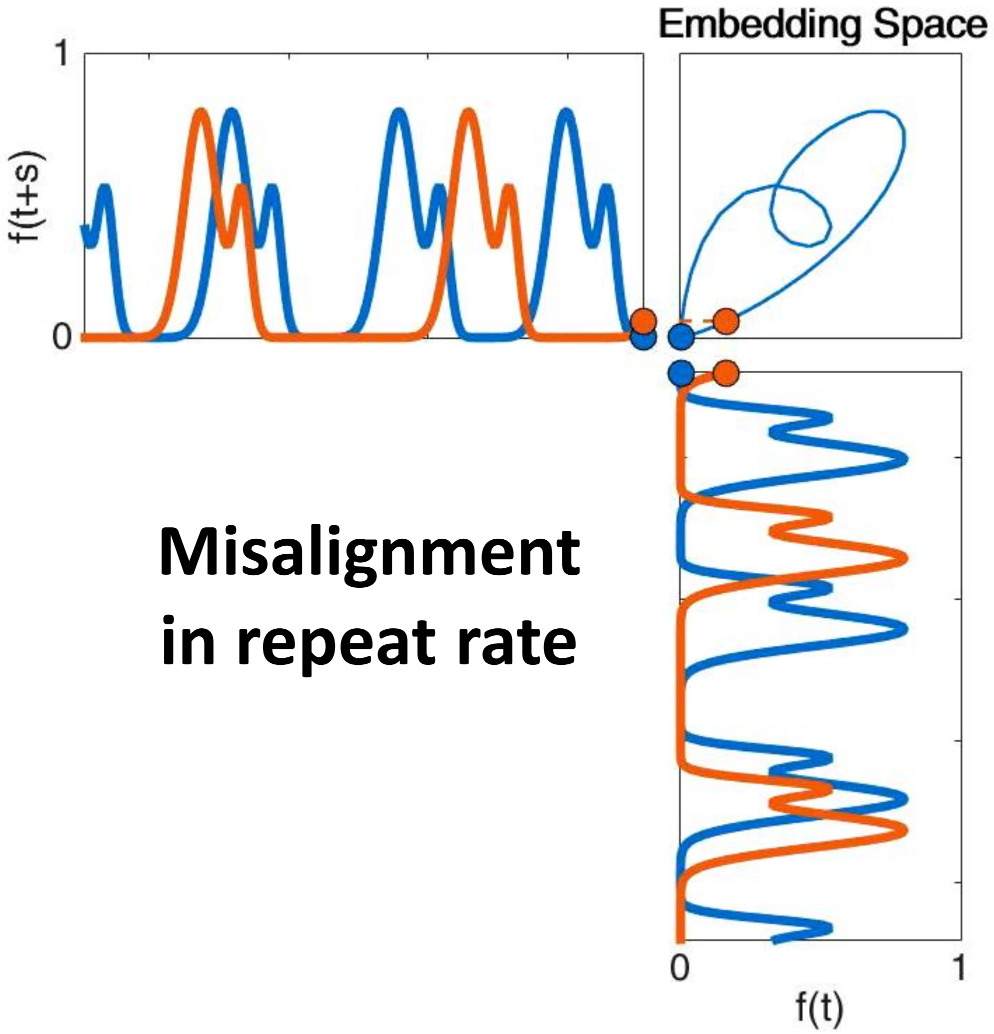
Φ --- estimate of x
s --- delay step
d --- embedding dimension
x_t --- state of the latent dynamical system at the time t
y_t --- observation (time series) at the time t

The infinite time series becomes a trajectory in a bounded embedding space. It performs in real time.

Invariance to Misalignment



Misalignment in phase

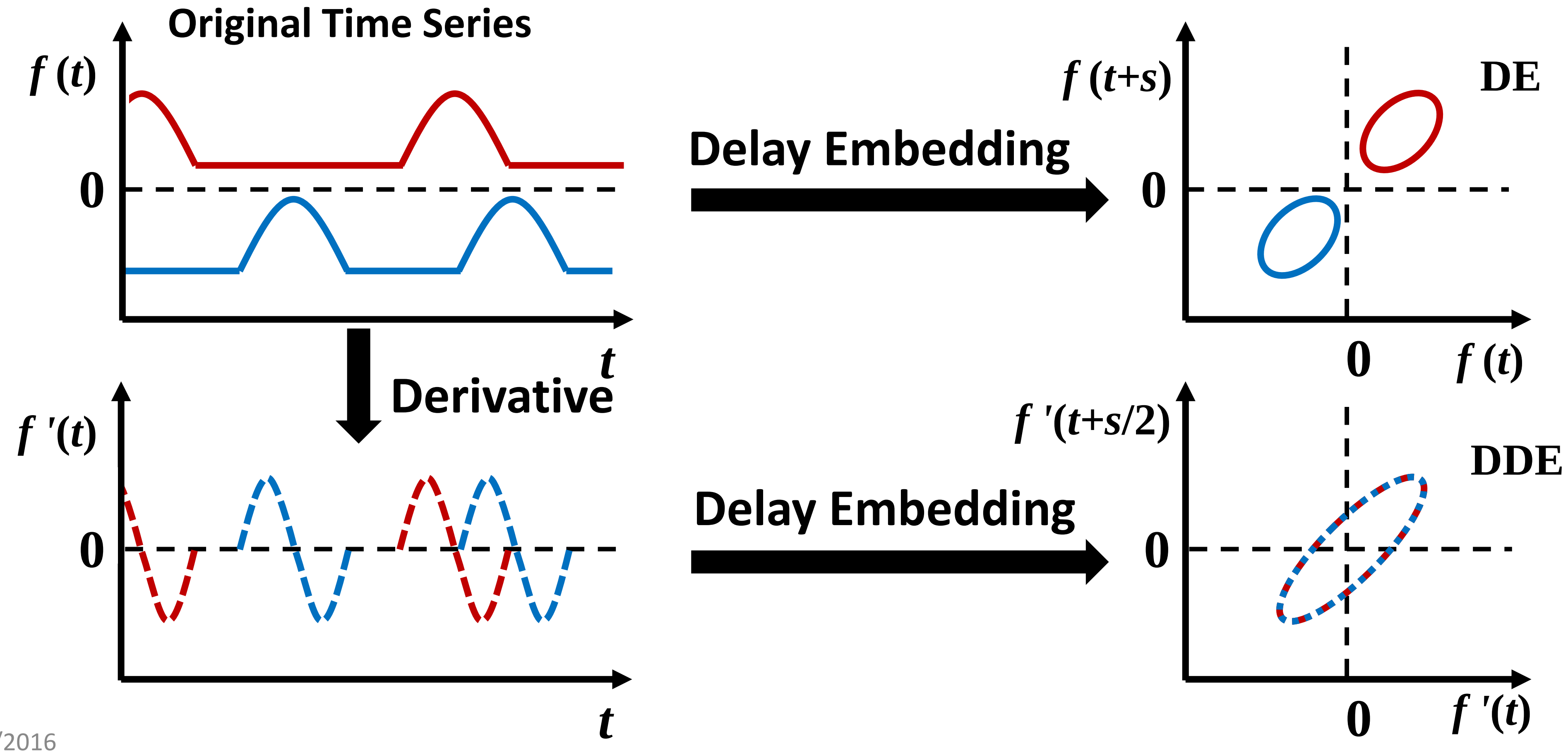


Misalignment in repeat rate

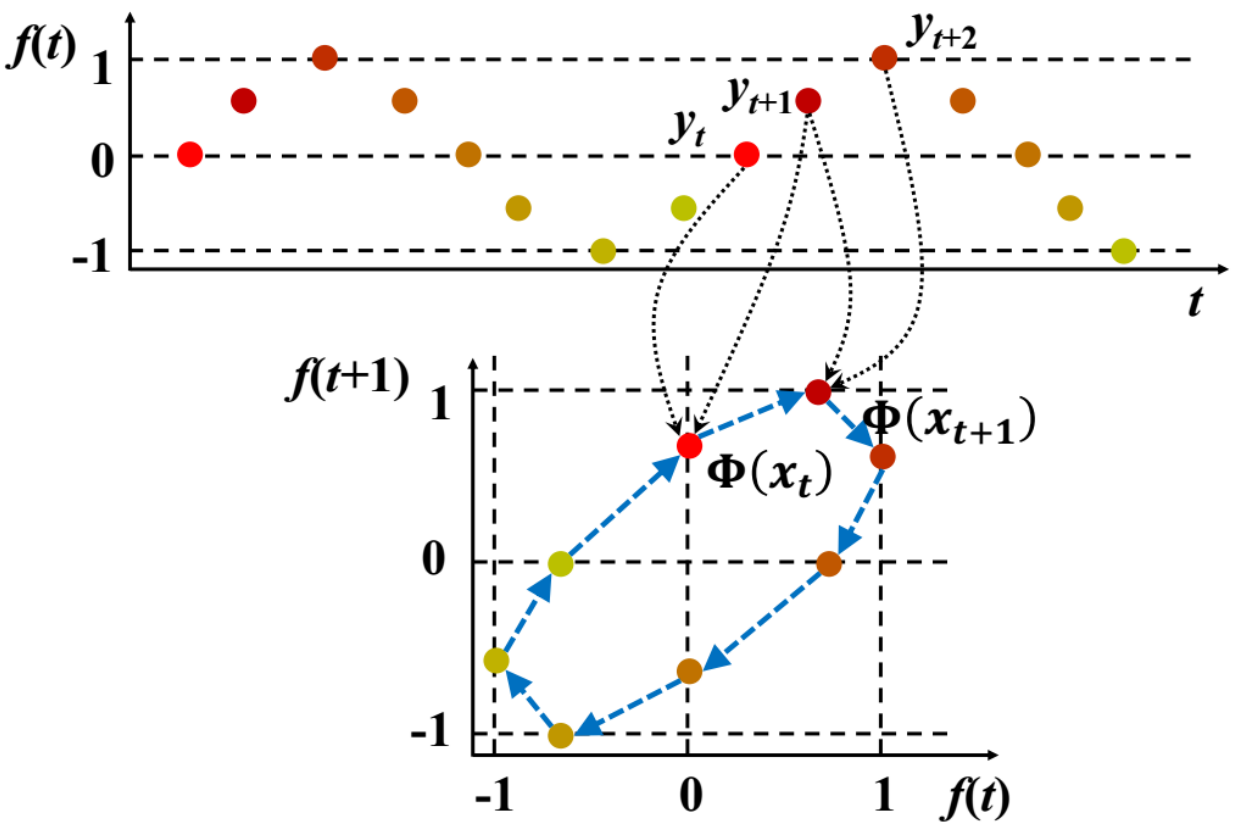
Misaligned streaming time series generate the same trajectory in the embedding space

DE → Derivative Delay Embedding (DDE)

Invariant to misalignment of baseline



Trajectory Modeling



In the embedding space, location of the states, and transition from one state to another carry the pattern of a trajectory.

Non-parametric model

- Probability the a state appear at certain location $P(x_t)$
- Transition probability $P(x_t \mid x_{t-1})$
- Discretized embedding space

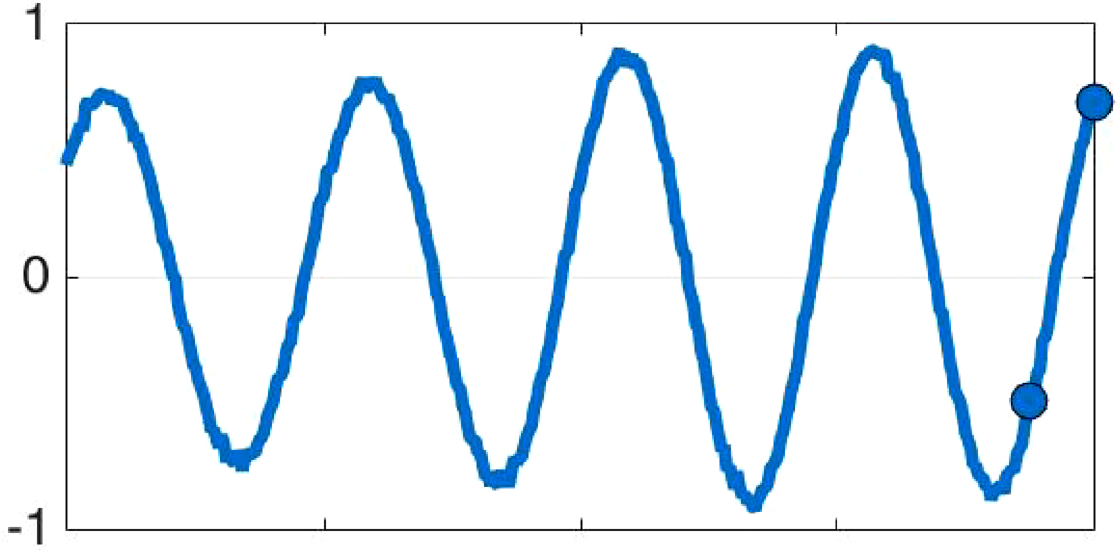
$$S_{\mathrm{MGM}}(X) = \sum_{j=1}^{t} P(x_j) \prod_{i=2}^{t} P(x_i|x_{i-1})$$
$$= S_{\mathrm{G}}(X) \times S_{\mathrm{M}}(X)$$

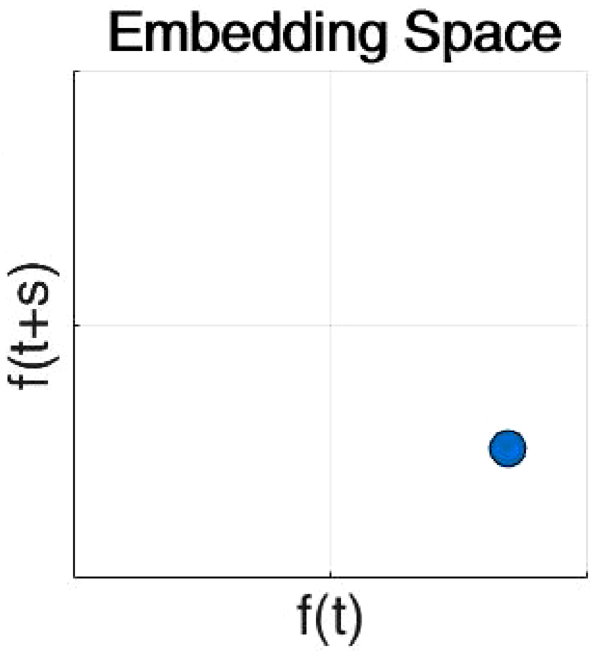
Markov Geographic Model (MGM)

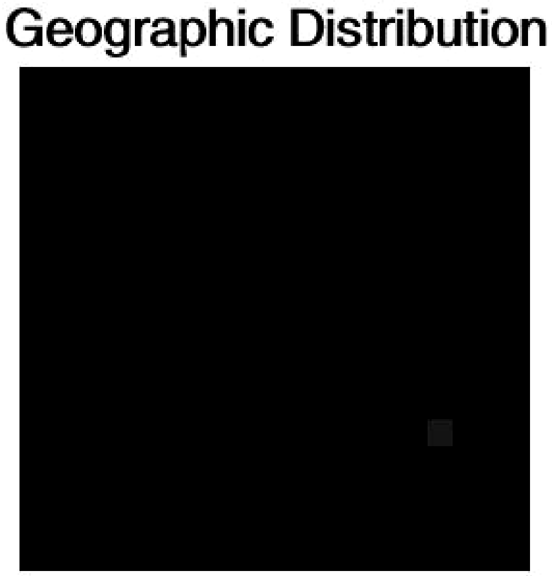
Online update the transition and distribution of states.

Markov process ← transition

Geographic distribution ← distribution



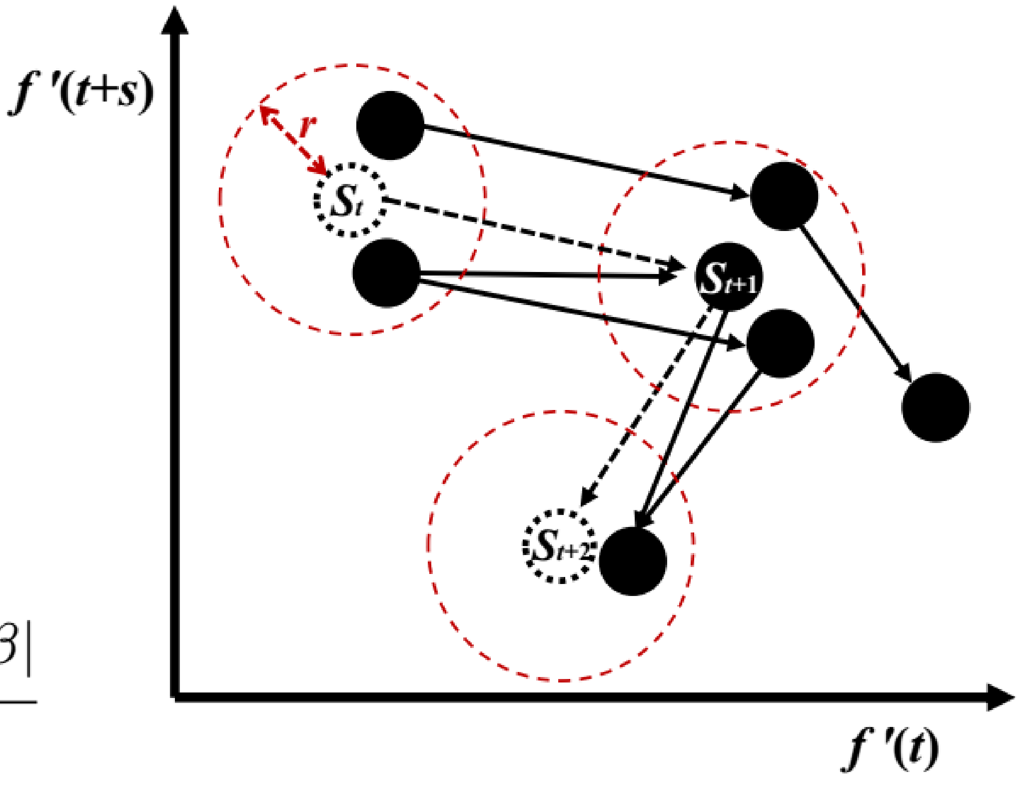




Neighborhood Matching

$$S_{\mathrm{MGM}}(X) = \sum_{j=1}^{t} P(x_j) \prod_{i=2}^{t} P(x_i | x_{i-1})$$
$$= S_{\mathrm{G}}(X) \times S_{\mathrm{M}}(X)$$

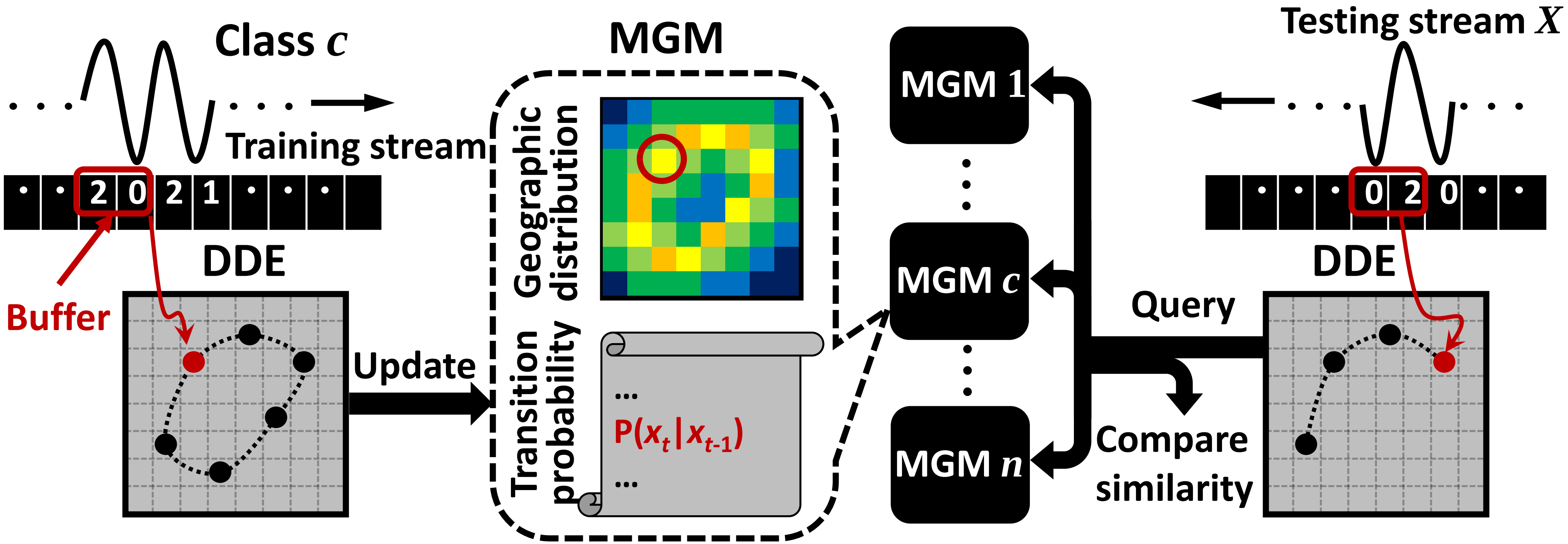
Make the transition probability more robust to noise and unseen samples in testing.



$$S_{\mathrm{M}}(X) = \prod_{i=2}^{t} \frac{\sum_{\alpha \in N_r(\Phi'(x_i)),\ \beta \in N_r(\Phi'(x_{i-1}))} |\alpha; \beta|}{\sum_k \sum_{\gamma \in N_r(\Phi'(x_{i-1}))} |\Phi'(x_k); \gamma|}$$

$N_r(\Phi'(x_i))$ --- the set of neighbors within radius r around $\Phi'(x_i)$

Online Modeling and Classification by DDE-MGM



Experimental Results

Datasets:

- UCI Character Trajectory --- 2858 character samples of 20 classes, x and y axes were recorded.

Normalized
Well aligned

- MSR Action3D --- 567 action samples of 20 classes performed by 10 subjects, human skeleton is recorded.

Misaligned
Variant length

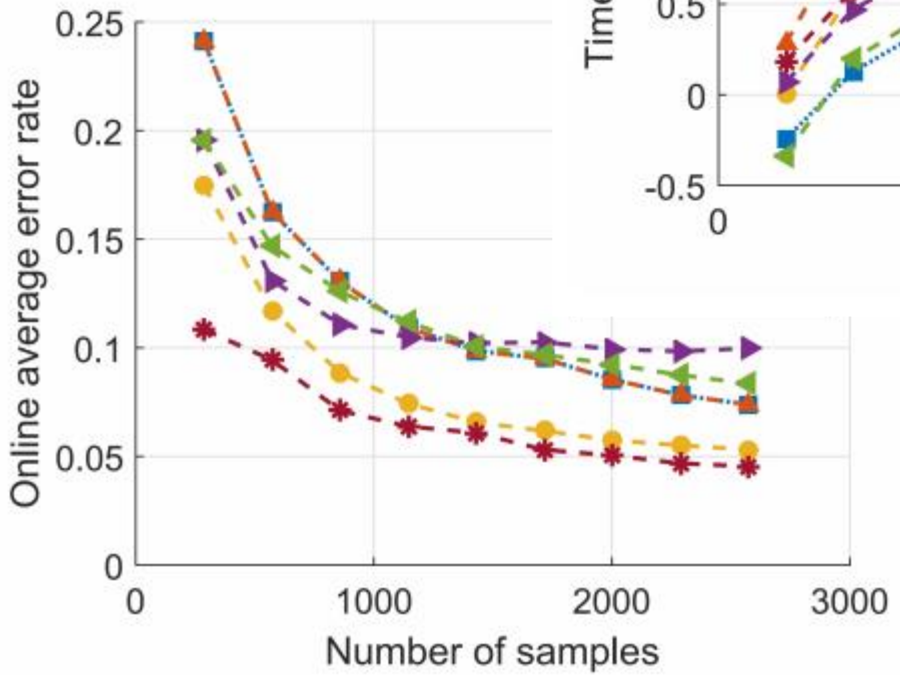
Experimental Results --- UCI Character Trajectory

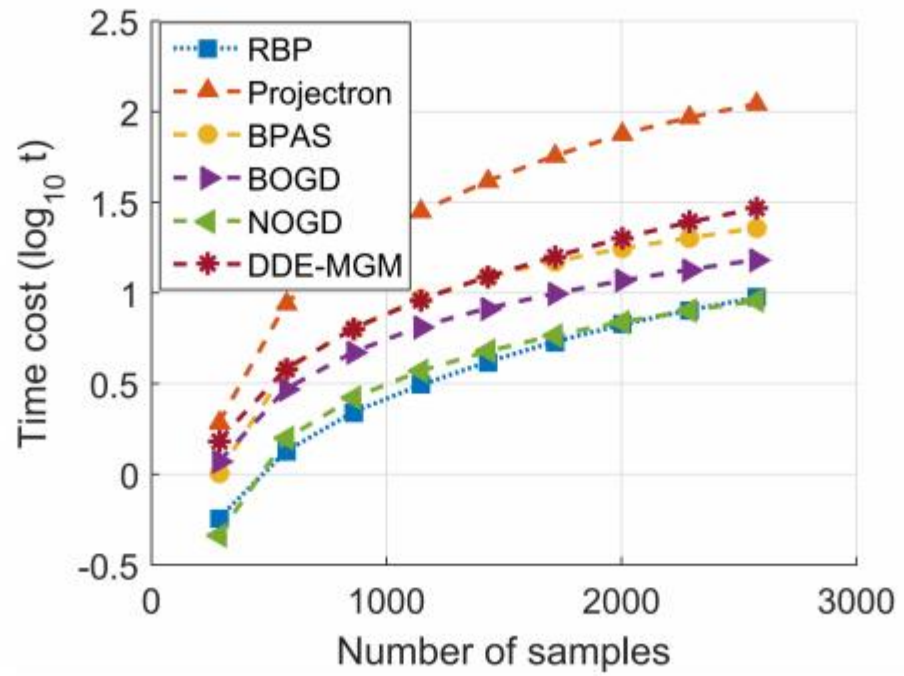
The data is normalized and well aligned

Half-vs-half validation

Algorithm	Accu. (%)	Time (sec)
1NN-DTW	91.37	3.9×10^4
SAX	89.96	128.85
HMM	57.89	7.4×10^3
DDE-MGM	**92.07**	**34.21**
RBP	92.62	9.44
Projectron	92.62	110.26
BPAS	94.68	22.81
BOGD	90.02	15.24
NOGD	91.65	**9.04**
DDE-MGM	**95.45**	63.92

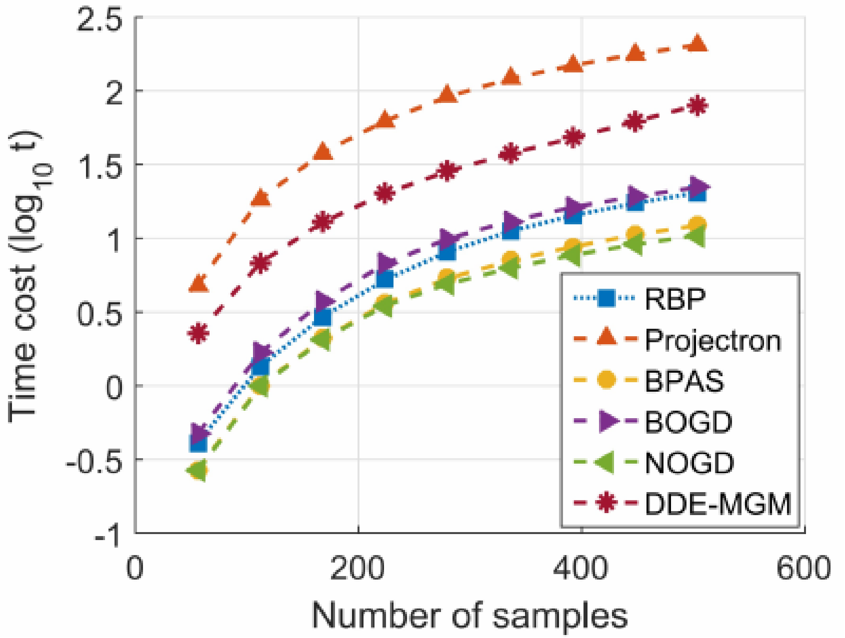
Online testing





Experimental Results --- MSR Action3D

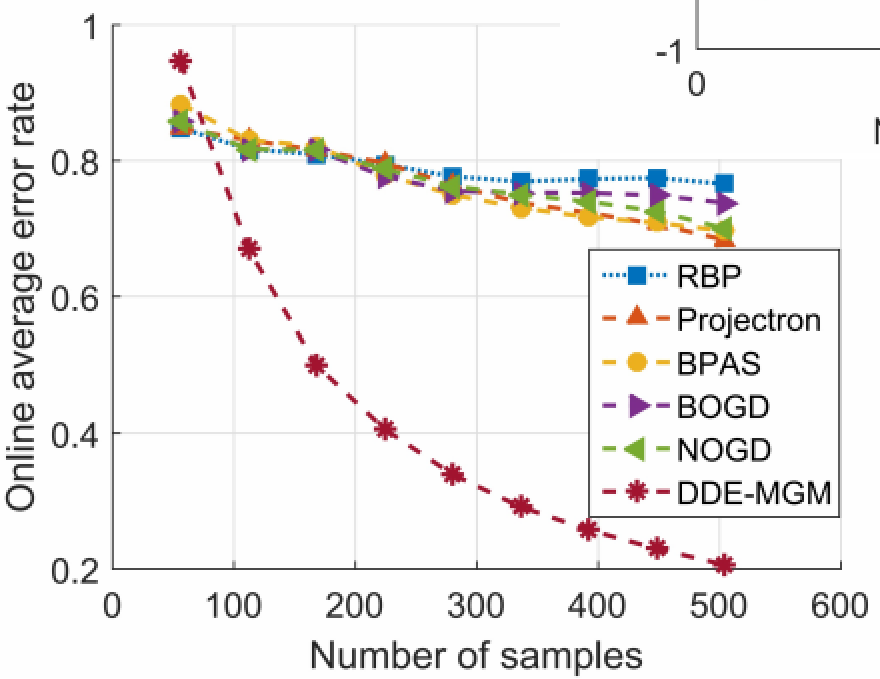
The data is not normalized and misaligned



Half-vs-half validation

Algorithm	Accu. (%)	Time (sec)
1NN-DTW	74.73	7.6×10^4
SAX	61.90	54.68
HMM	60.07	2.1×10^3
DDE-MGM	**93.04**	**28.40**
RBP	23.41	20.23
Projectron	31.65	205.25
BPAS	30.36	12.25
BOGD	26.19	22.23
NOGD	29.96	**10.47**
DDE-MGM	**79.37**	80.38

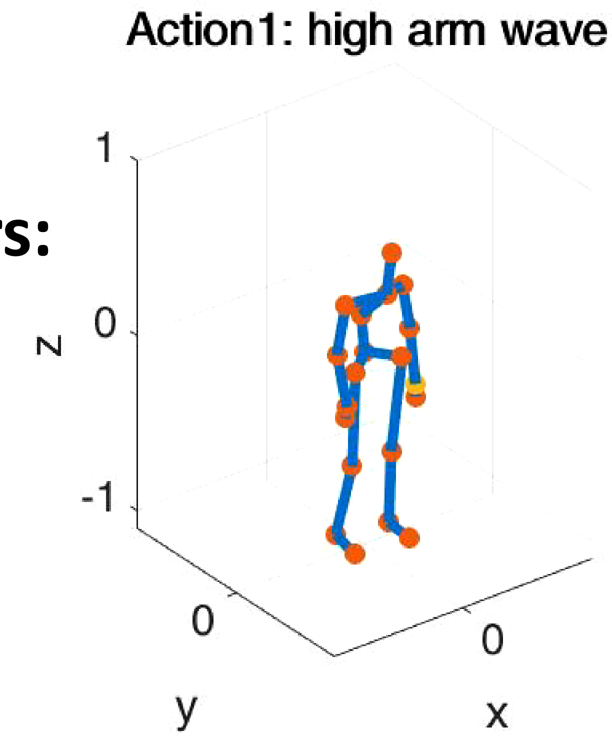
Online testing

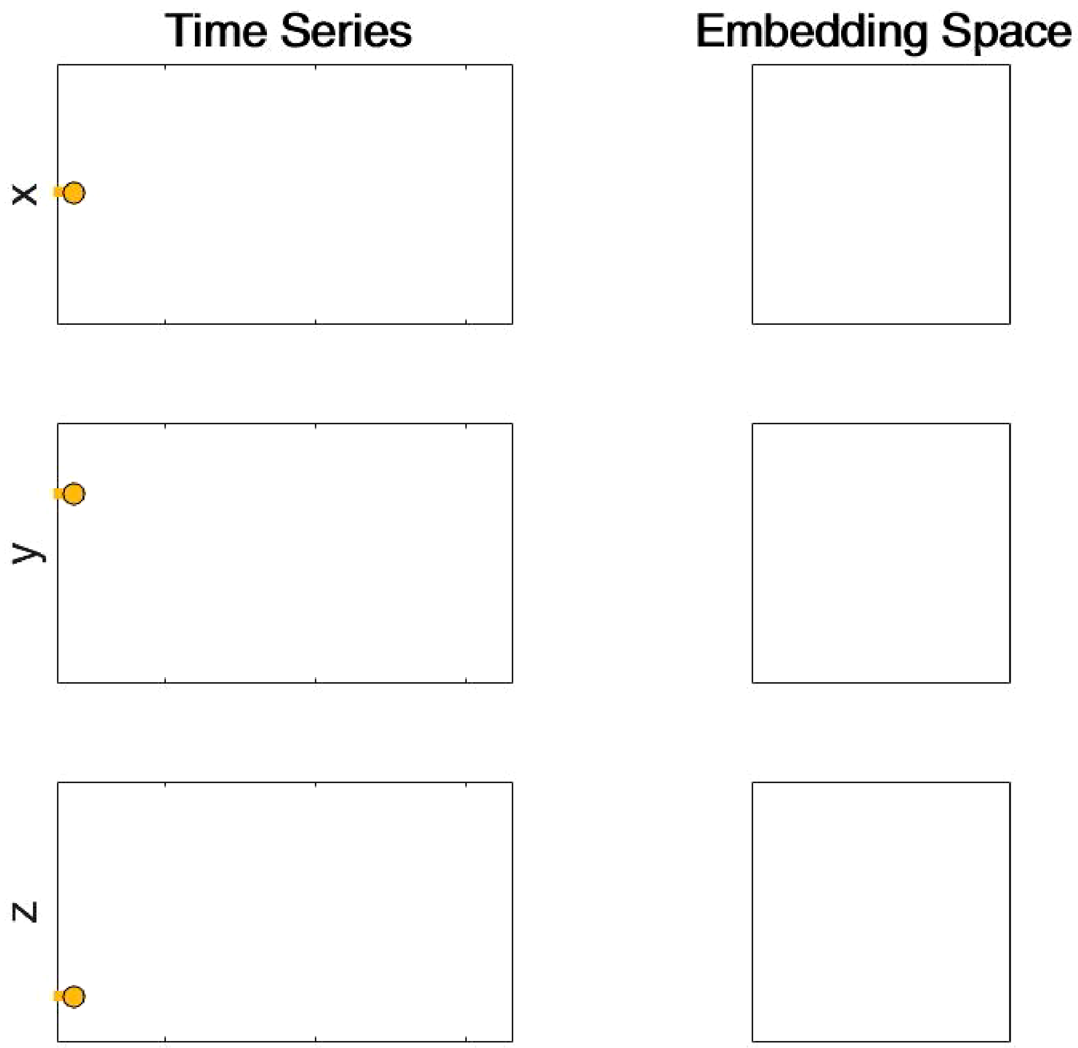


An Example of Action Recognition

The joint of left wrist is plotted for three categories of actions shown in different colors:

- high arm wave
- horizontal arm wave
- hammer





Conclusion

- **DDE is introduced to solve misalignment in online modeling**
- **The non-parametric model MGM is proposed to model the trajectories in an online manner**
- **Both modeling and classification are achieved in real time.**

THANKS

SIGIR
Special Interest Group
on Information Retrieval

Student Travel Grant

Appendix: Parameter Setting of DE

$$\Phi(x_t; s, d) = (y_t, y_{t+s}, \cdots, y_{t+(d-1)s})$$

Φ --- estimate of x

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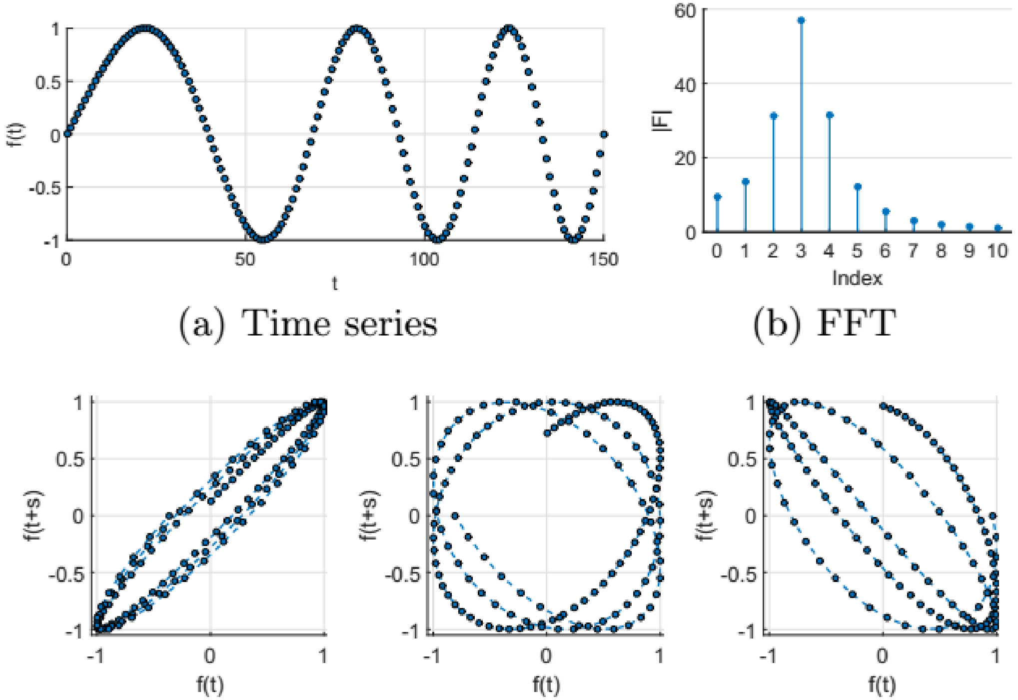

(a) Time series (b) FFT

(c) $s = 2$ (d) $s = 12$ (e) $s = 25$

d --- False nearest neighbor [M. Kennel et al., 1992]

s --- $2\pi \times d \times s \times \frac{f}{f_s} \equiv 0 \mod \pi$ [J. A. Perea and J. Harer, 2013]